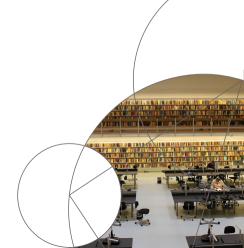


Causal Inference

Sebastian Weichwald

🕆 sweichwald.de 灯 @sweichwald



Learn more on Causality at Lviv Summer School 2020

- Fundamentals of Causal Learning
- Wednesday, July 22
- 8:00-12:30 UTC
- Marharyta Aleksandrova



More online lectures on Causality

- 4 lectures on causality by J Peters (8 h)
 MIT Statistics and Data Science Center, 2017 stat.mit.edu/news/four-lectures-causality
- causality tutorial by D Janzing and S Weichwald (4 h)
 Conference on Cognitive Computational Neuroscience 2019 sweichwald.de/ccn2019
- course on causality by S Bauer and B Schölkopf (3 h)
 Machine Learning Summer School 2020 youtube.com/watch?v=btmJtThWmhA
- course on causality by D Janzing and B Schölkopf (3 h)
 Machine Learning Summer School 2013 mlss.tuebingen.mpg.de/2013/speakers.html





instrumental variables consistency and asymptotic normality number of variables false discovery illustrate the performance theoretical findings reproducing kernel Hilbert space semi-supervised learning real-world data structural equation models efficient estimation confidence intervals

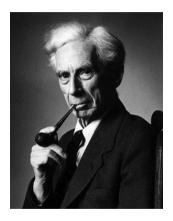
mercular predictive performance finite sample maximum likelihood

MERLIN antipode joint distribution hidden variables

maximum likelihood estimator brain activity asymptotic normality variable selection maximum likelihood estimator brain activity asymptotic normality finite mixture cortical confounding regression model edge weights across different nonparametric intervention additive noise structure across different nonparametric Hilbert-Schmischen et al. (1988) and the second additive noise structure equations additive noise structure equations additive noise structure equations additive noise structure equations across different nonparametric Hilbert-Schmischen et al. (1988) and the second noise structure equations across different expenses and the second property brain undirected brain undirected brain undirected second proposed in the metric spaces
time series
sparsity
multivariate
sponential family
tensafer learning decoding directed acyclic graphs
observed variables
conditional distribution Bayesian networks
metric spaces
sparsity
sparsity totally positive likelihood ratio encoding and decoding network model parameter estimation regression coefficients undirected graph time series models explanatory variables independent component analysis

high-dimensional estimating equations





"All philosophers, of every school, imagine that causation is one of the fundamental axioms or postulates of science, yet, oddly enough, in advanced sciences such as gravitational astronomy, the word "cause" never occurs. [...] To me, it seems that [...] the reason why physics has ceased to look for causes is that, in fact, there are no such things. The law of causality, I believe, like much that passes muster among philosophers, is a relic of a bygone age, surviving, like the monarchy, only because it is erroneously supposed to do no harm."

- B Russell (1913), On the Notion of Cause



Image from theeconplayground.com.

Sebastian Weichwald — Causal Inference — Slide 5



"Fortunately, very few physicists paid attention to Russell's enigma. They continued to write equations in the office and talk cause–effect in the cafeteria; with astonishing success they smashed the atom, invented the transistor and the laser.

The same is true for engineering."

- J Pearl (2009), Causality





Causal questions require causal answers.



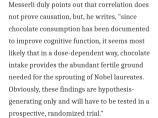


Eating chocolate produces Nobel prize winners, says study

By Oliver Nieburg 37, 11-Oct-2012

Related tags: noble prize, nobel laureate, Einstein, Marie Curie, chocolate, brain, Switzerland, Sweden, candy







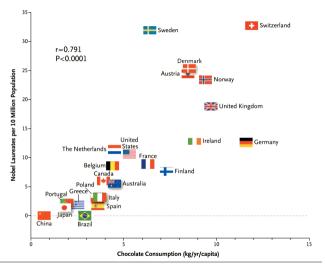
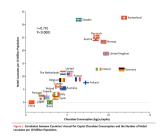
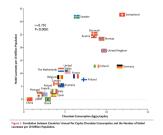


Figure 1. Correlation between Countries' Annual Per Capita Chocolate Consumption and the Number of Nobel Laureates per 10 Million Population.



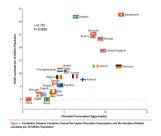






Kim goes on a cruise to another country..

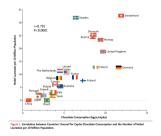




Kim goes on a cruise to another country..

SEEING: ..and reports back that year's chocolate consumption.



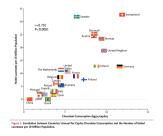


Kim goes on a cruise to another country..

SEEING: ..and reports back that year's chocolate consumption.

DOING: ..and brings enormous amounts of chocolate for a year.





Kim goes on a cruise to another country..

SEEING: ..and reports back that year's chocolate consumption.

DOING: ..and brings enormous amounts of chocolate for a year.



→ Can we predict #country's Nobel Laureates?

 ${\bf Z}$ Causal questions require causal answers.



? Causal questions require causal answers.

© "Correlation does not imply causation."

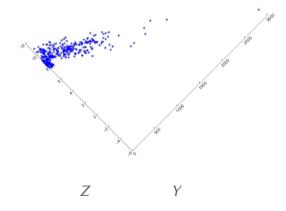


? Causal questions require causal answers.

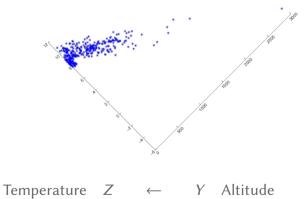
© "Correlation does not imply causation."

SEEING VS DOING

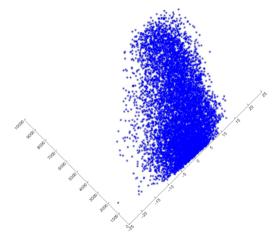




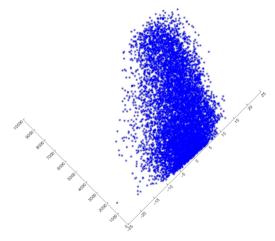














? Causal questions require causal answers.

© "Correlation does not imply causation."

SEEING VS DOING



- ? Causal questions require causal answers.
- © "Correlation does not imply causation."

SEEING VS DOING

© Correlation(s) may tell us something about causation.



- ? Causal questions require causal answers.
- © "Correlation does not imply causation."

SEEING VS DOING

- Ocrrelation(s) may tell us something about causation.
- → Causal Inference: assumptions, data, explicit, algorithmic



CAUSATION AS A PROGRAMMER'S NIGHTMARE

Input:

- 1. "If the grass is wet, then it rained"
- 2. "If we break this bottle, the grass will get wet"

Output:

"If we break this bottle, then it rained"



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NEEDED: ALGEBRA OF DOING

Available: algebra of seeing

e.g., What is the chance it rained if we see the grass wet?

$$P(rain \mid wet) = ?$$
 $\{= P(wet \mid rain) \frac{P(rain)}{P(wet)}\}$

Needed: algebra of doing

e.g., What is the chance it rained if we make the grass wet?

$$P(rain \mid do(wet)) = ? \{= P(rain)\}$$



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Formalizing the difference between seeing and doing

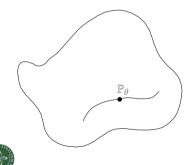
- **observational probabilities:** p(y|x) probability for Y = y, given that we observed X = x
- interventional probabilities: p(y|do(x)) probability for Y = y, given that we have set X to x

Confusing p(y|x) with p(y|do(x)) is the reason for most of the common misconceptions about causality!

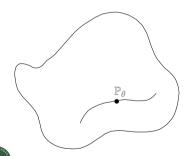


"Normal" Probabilistic Model:

$$\mathcal{M}_X:\theta\mapsto\mathbb{P}_\theta$$

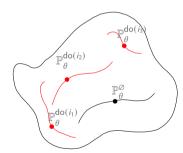


$$\mathcal{M}_X:\theta\mapsto\mathbb{P}_\theta$$

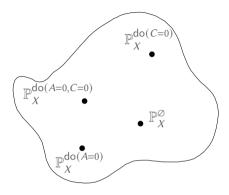


Causal Model:

 $\mathcal{M}_X: \theta \mapsto \{\mathbb{P}^{\mathsf{do}(i)}_{\theta} : i \in I_X\}$ I_X is set of interventions.

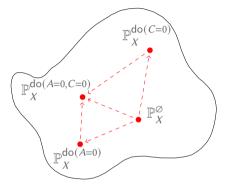


Causal Models





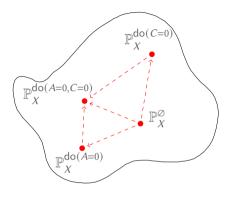
Causal Models



 I_X has partial ordering structure



Causal Models



 I_X has partial ordering structure



 \mathcal{M}_X implies the poset of distributions $\mathcal{P}_X := \left(\left\{ \mathbb{P}_X^{\mathsf{do}(i)} : i \in \mathcal{I}_X \right\}, \leq_X \right)$

Structural Causal Models

$$\mathcal{M}_X = (\mathcal{S}_X, \mathcal{I}_X, \mathbb{P}_{E_X})$$



Structural Causal Models

$$\mathcal{M}_X = (\mathcal{S}_X, \mathcal{I}_X, \mathbb{P}_{E_X})$$

•
$$S_X = \begin{cases} X_1 = E_1 \\ X_2 = X_1 + E_2 \end{cases}$$



Structural Causal Models

$$\mathcal{M}_X = (\mathcal{S}_X, \mathcal{I}_X, \mathbb{P}_{E_X})$$

•
$$S_X = \begin{cases} X_1 = E_1 \\ X_2 = X_1 + E_2 \end{cases}$$

• $I_X = \{ \emptyset, \operatorname{do}(X_1 = 5), \operatorname{do}(X_2 = 3) \}$



$$\mathcal{M}_X = (\mathcal{S}_X, \mathcal{I}_X, \mathbb{P}_{E_X})$$

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- $E \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$



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• $E \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$

observational

$$\mathbb{P}_{X_1}^{\varnothing} \sim \mathcal{N}(0,1)$$

$$\mathbb{P}_{X_2}^{\varnothing} \sim \mathcal{N}(0,2)$$



$$\mathcal{M}_{X} = (\mathcal{S}_{X}, \mathcal{I}_{X}, \mathbb{P}_{E_{X}})$$

$$\bullet \quad \mathcal{S}_{X} = \begin{cases} X_{1} = E_{1} \\ X_{2} = X_{1} + E_{2} \end{cases}$$

$$\bullet \quad \mathcal{I}_{X} = \{\emptyset, \operatorname{do}(X_{1} = 5), \operatorname{do}(X_{2} = 3)\}$$

$$\bullet \quad E \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

observational

intervention on X_1

$$\mathbb{P}_{X_1}^{\varnothing} \sim \mathcal{N}(0,1)$$
 $\mathbb{P}_{X_1}^{\mathsf{do}(X_1=5)} \equiv 5$

$$\mathbb{P}_{X_2}^{\varnothing} \sim \mathcal{N}(0,2)$$
 $\mathbb{P}_{X_2}^{\mathsf{do}(X_1=5)} \sim \mathcal{N}(5,1)$



$$\mathcal{M}_{X} = (\mathcal{S}_{X}, \mathcal{I}_{X}, \mathbb{P}_{E_{X}})$$

$$\bullet \quad \mathcal{S}_{X} = \begin{cases} X_{1} = E_{1} \\ X_{2} = X_{1} + E_{2} \end{cases}$$

$$\bullet \quad \mathcal{I}_{X} = \{\emptyset, \operatorname{do}(X_{1} = 5), \operatorname{do}(X_{2} = 3)\}$$

$$\bullet \quad E \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

observational

intervention on X_1

intervention on X_2

$$\mathbb{P}_{X_1}^{\varnothing} \sim \mathcal{N}(0,1)$$

$$\mathbb{P}_{X_1}^{\mathsf{do}(X_1=5)} \equiv 5$$

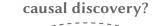
$$\mathbb{P}_{X_1}^{\mathsf{do}(X_2=3)} \sim \mathcal{N}(0,1)$$

$$\mathbb{P}_{X_2}^{\varnothing} \sim \mathcal{N}(0,2)$$

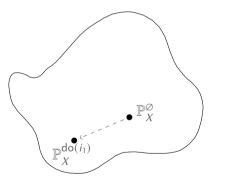
$$\mathbb{P}_{X_2}^{\mathsf{do}(X_1=5)} \sim \mathcal{N}(5,1)$$

$$\mathbb{P}_{X_2}^{\mathsf{do}(X_2=3)} \equiv 3$$

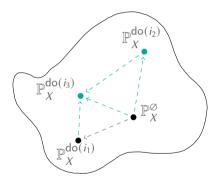






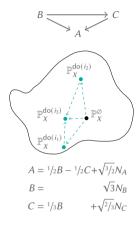


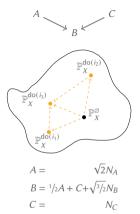
$$\left\{ \mathbb{P}_{X}^{\mathsf{do}(i)}:\ i\in\mathcal{I}_{\mathsf{sub}}\subsetneq\mathcal{I}_{X}\right\}$$

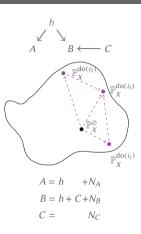


causal model

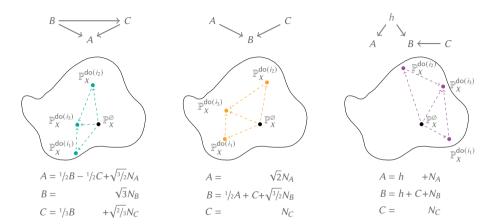




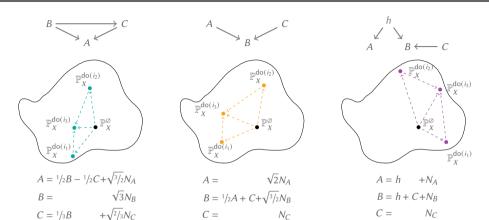








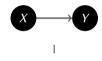
→ 3 models inducing the same observational yet different interventional distributions



→ 3 models inducing the same observational yet different interventional distributions

£ fitting observational data well is not enough £

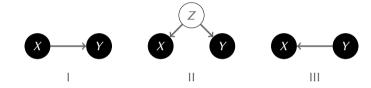






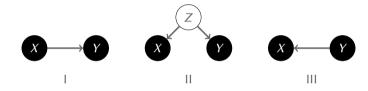






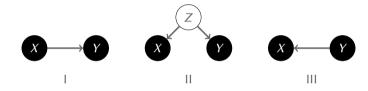


If two variables *X* and *Y* are statistically dependent then either



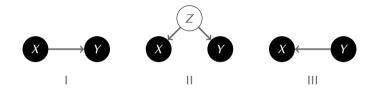
every statistical dependence is due to a causal relation





- every statistical dependence is due to a causal relation
- cases I, II, and III can also occur simultaneously



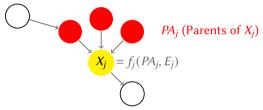


- every statistical dependence is due to a causal relation
- cases I, II, and III can also occur simultaneously
- distinction between the 3 cases is a key problem in scientific reasoning



Functional model of causality Pearl and Spirtes, Glymour, Scheines

• every node X_j is a function of its parents and an unobserved noise term E_j

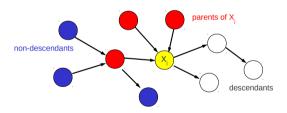


- all noise terms E_i are statistically independent (causal sufficiency)
- which properties of $P(X_1, ..., X_n)$ follow?



Causal Markov condition (4 equivalent statements)

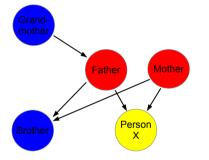
- existence of a functional model
- **local Markov condition:** every node is conditionally independent of its non-descendants, given its parents





- global Markov condition: describes all independences via d-separation
- Markov factorization: $P(X_1, ..., X_n) = \prod_i P(X_i | PA_i)$

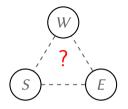
Metaphor for the local Markov condition

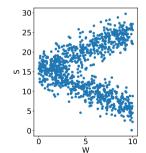


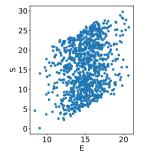
If someone knows the genes of *X*'s parents, neither the genes of the grandmother nor the genes of the brother contain additional information about *X*.

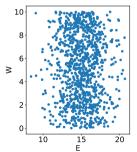


Given observations of W, E, and S (5%), what is the causal structure?



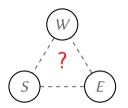




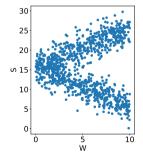


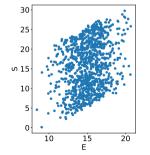


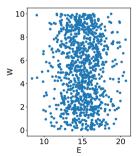
Given observations of W, E, and S (\mathcal{S}_{\bullet}), what is the causal structure?



• Which variable pairs are (in)dependent? (All dependent given the 3rd.)

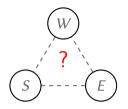




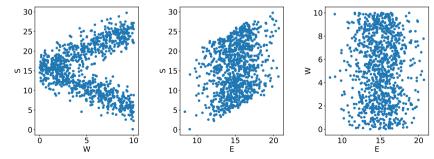




Given observations of W, E, and S (5), what is the causal structure?

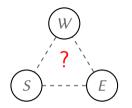


- Which variable pairs are (in)dependent? (All dependent given the 3rd.)
- Assume existence of a functional model.

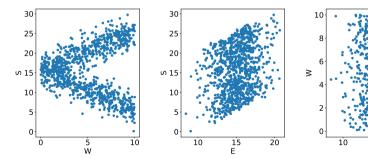




Given observations of W, E, and S (5), what is the causal structure?



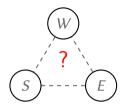
- Which variable pairs are (in)dependent? (All dependent given the 3rd.)
- Assume existence of a functional model.
- Which causal structures are possible? (7)



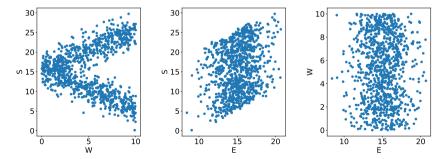


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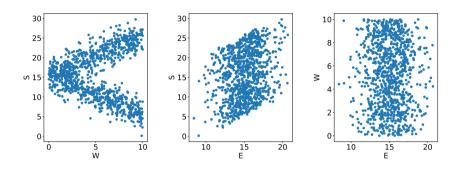
Given observations of W, E, and S (\mathfrak{S}), what is the causal structure?



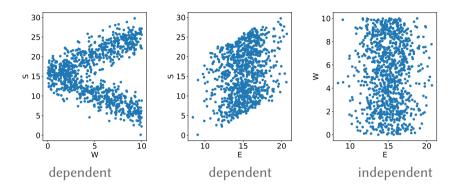
- Which variable pairs are (in)dependent? (All dependent given the 3rd.)
- Assume existence of a functional model.
- Which causal structures are possible? (7)
- Further assumption to narrow it down?



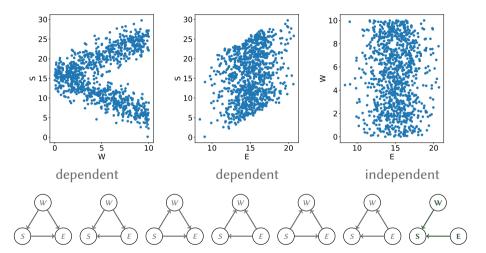




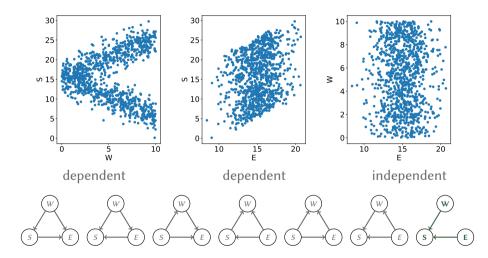














Assume existence of a functional model (causal sufficiency) & faithfulness.

Pearl's do-operator

How to compute $p(x_1, ..., x_n | do(x_i^*))$:

• Write $p(x_1, \ldots, x_n)$ as

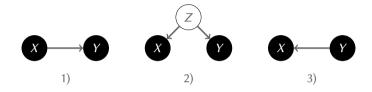
$$\prod_{k=1}^{n} p(x_k | \operatorname{parents}(x_k))$$

• and replace $p(x_i| \text{ parents}(x_i))$ with δ_{x_i,x_i^*}

$$p(x_1,\ldots,x_n|\operatorname{do}(x_i^*))=\delta_{x_i,x_i^*}\prod_{k\neq i}p(x_k|\operatorname{parents}(x_k))$$



Examples



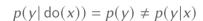
• interventional and observational probabilities coincide (seeing = doing)

$$p(y|\operatorname{do}(x)) = p(y|x)$$

2 intervening on *x* does not change *y*

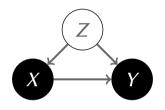
$$p(y|\operatorname{do}(x)) = p(y) \neq p(y|x)$$

3 intervening on x does not change y





Confounder correction



$$p(y|\operatorname{do}(x)) = \sum_{z} p(y|x,z)p(z) \neq \sum_{z} p(y|x,z)p(z|x) = p(y|x)$$



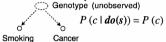
COPENHAGEN CAUSALITY LAB

SMOKING AND CANCER: HANDLING COMPETING MODELS

1. Surgeon General (1964):

$$\bigcirc$$
 $P(c \mid do(s)) \approx P(c \mid s)$
Smoking Cancer

2. Tobacco Industry:



3. Combined:

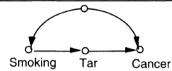
$$P(c \mid do(s)) = \text{noncomputable}$$
Smoking Cancer

4. Combined and refined:

$$P(c \mid do(s)) = \text{computable}$$
Smoking Tar Cancer



TYPICAL DERIVATION IN CAUSAL CALCULUS!

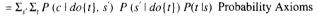


$$P(c \mid do\{s\}) = \sum_{s} P(c \mid do\{s\}, t) P(t \mid do\{s\})$$
 Probability Axioms

$$= \sum_{t} P(c \mid do\{s\}, do\{t\}) P(t \mid do\{s\})$$

$$= \sum_t P\left(c \mid do\{s\}, do\{t\}\right) P\left(t \mid s\right)$$

$$= \sum_{t} P\left(c \mid do\{t\}\right) P\left(t \mid s\right)$$



$$= \sum_{s'} \sum_{t} P(c \mid t, s') P(s' \mid do\{t\}) P(t \mid s)$$



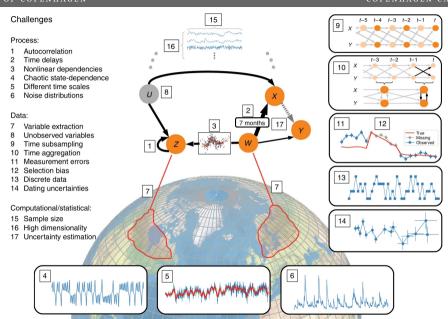
$$\mathbf{47} = \Sigma_{s'} \Sigma_{t} P(c \mid t, s') P(s') P(t \mid s)$$





Level	Typical	Typical Questions	Examples
(Symbol)	Activity		
1. Association	Seeing	What is?	What does a symptom tell me
P(y x)		How would seeing X	about a disease?
		change my belief in Y ?	What does a survey tell us
			about the election results?
2. Intervention	Doing	What if?	What if I take aspirin, will my
P(y do(x),z)	Intervening	What if I do X ?	headache be cured?
			What if we ban cigarettes?
3. Counterfactuals	Imagining,	Why?	Was it the aspirin that
$P(y_x x',y')$	Retrospection	Was it X that caused Y ?	stopped my headache?
		What if I had acted	Would Kennedy be alive had
		differently?	Oswald not shot him?
			What if I had not been smok-
			ing the past 2 years?







 ${\it 1}$ Causal questions require causal answers.



Causal questions require causal answers.

© "Correlation does not imply causation."



? Causal questions require causal answers.

© "Correlation does not imply causation."

SEEING VS DOING



- ? Causal questions require causal answers.
- © "Correlation does not imply causation."

SEEING VS DOING

© Correlation(s) may tell us something about causation.



- ? Causal questions require causal answers.
- © "Correlation does not imply causation."

SEEING VS DOING

- © Correlation(s) may tell us something about causation.
- → Causal Inference: assumptions, data, explicit, algorithmic



Causal Inference: assumptions, data, explicit, algorithmic









Causal Inference: assumptions, data, explicit, algorithmic

- 4 lectures on causality by J Peters (8 h) MIT Statistics and Data Science Center, 2017 stat.mit.edu/news/four-lectures-causality
- causality tutorial by D Janzing and S Weichwald (4 h) Conference on Cognitive Computational Neuroscience 2019 sweichwald.de/ccn2019
- course on causality by S Bauer and B Schölkopf (3 h) Machine Learning Summer School 2020 youtube.com/watch?v=btmJtThWmhA
- course on causality by D Janzing and B Schölkopf (3 h) Machine Learning Summer School 2013 mlss.tuebingen.mpg.de/2013/speakers.html







